

Selection and Mutation

23.10.2005

GE02: day 2 part 3

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Selection

Acts through differential reproduction

Fitness of genotype g

- probability of reproductive success of an individual having this genotype
- $r(g)$: “**r**eproductive success” or “survival”
- $s(g)$: “**S**election pressure” or “mortality”
- $r(g) = 1 - s(g)$

Differential reproduction

Can occur because of different mechanisms

- Survival, e.g. a mutation may lead to death before reproductive age
- Damaged reproductive system, e.g. complete sterility or lowered fertility
- Distorted sexual behavior
- ...

Problem: dominant lethal allele

- Two alleles are present in population, **N** and **D**
- Initial frequency of **D** is q_0
- Survival in **NN** is not affected (normal fitness), while carriers of **D** have absolute mortality (all dead at early development):
$$s(\mathbf{NN}) = 0, s(\mathbf{ND}) = s(\mathbf{DD}) = 1$$
- What will happen to the allele **D** after some time?

Solution

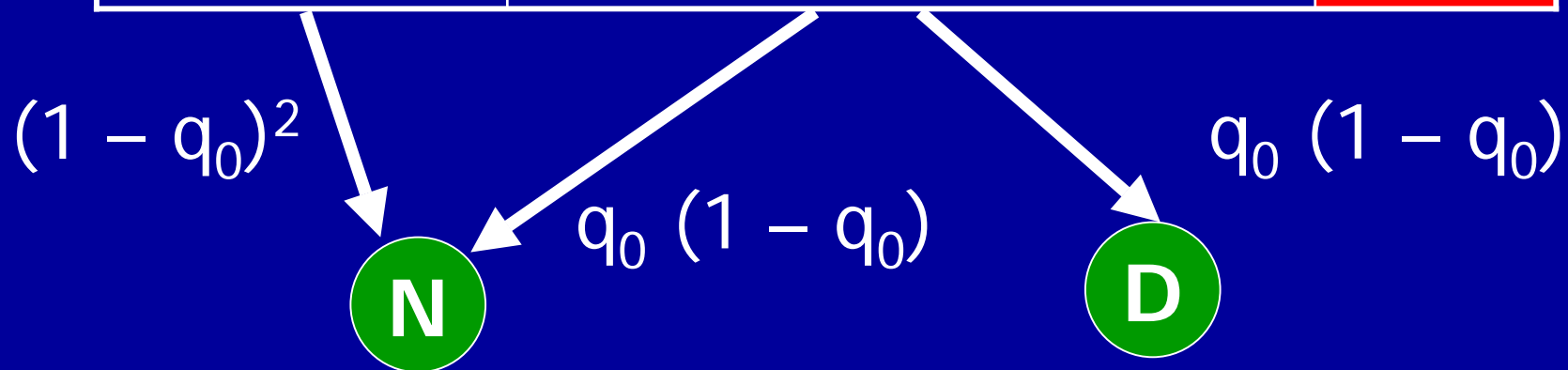
- It will be immediately eliminated!

Problem: recessive mutation

- Two alleles, **N** and **D**
- Initial frequency of **D** is q_0
- Population is large and mating is random
- Individuals of genotype **DD** are sterile, while fitness of other genotypes is not distorted:
$$s(\mathbf{NN}) = s(\mathbf{ND}) = 0; s(\mathbf{DD})=1$$
- What will happen to the allele **D** after some time?

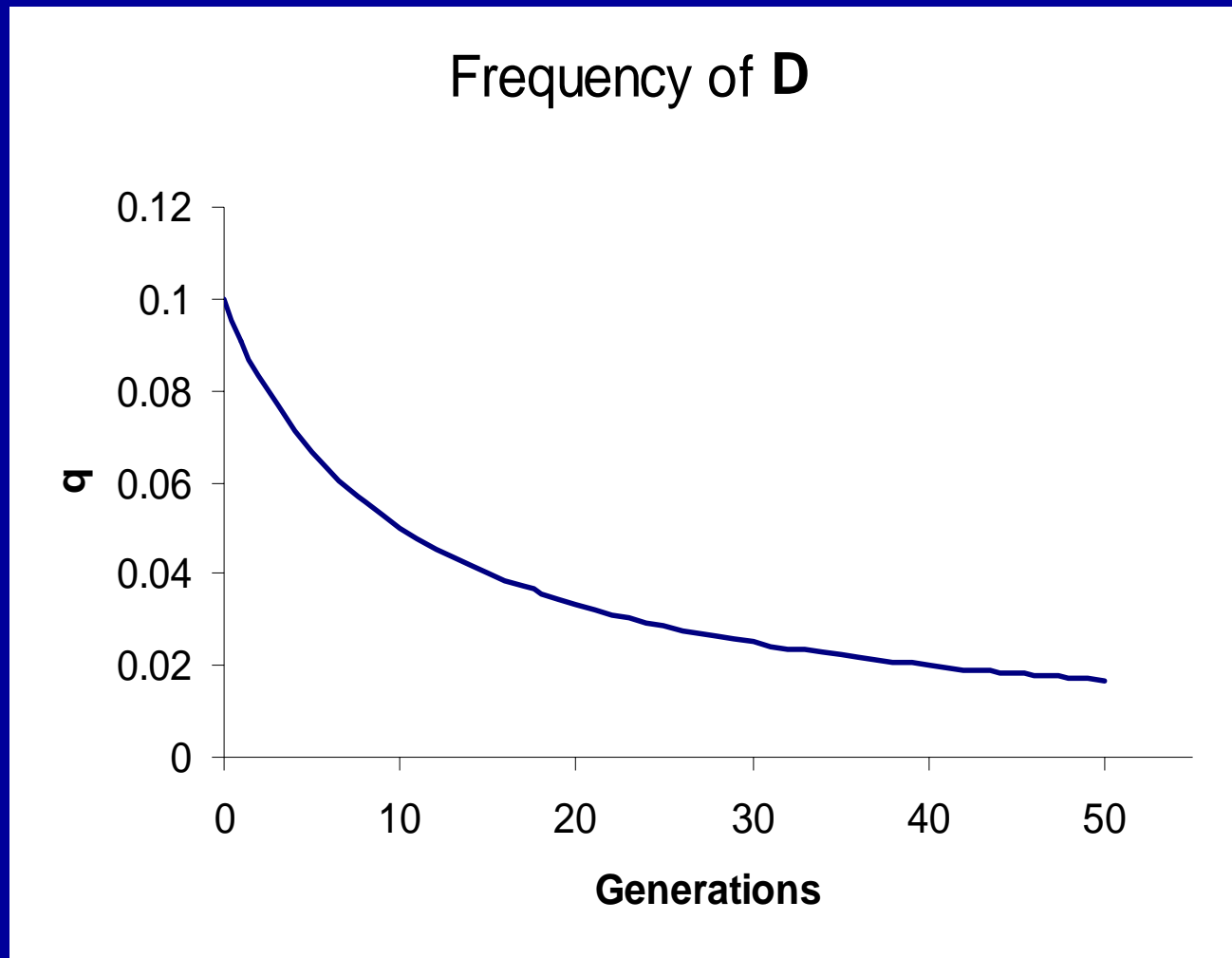
Solution

NN	ND	DD
$(1 - q_0)^2$	$2 q_0 (1 - q_0)$	q_0^2



$$q_1 = \frac{q_0(1 - q_0)}{(1 - q_0)^2 + 2q_0(1 - q_0)} = \frac{q_0}{(1 - q_0) + 2q_0} = \frac{q_0}{1 + q_0}$$

Elimination of mutant



Generalisation to arbitrary s

- Two alleles, **D** (deleterious) and **N** (normal)
- Survival in **DD** is decreased by some s :
 - only $(1 - s)$ survive and reproduce

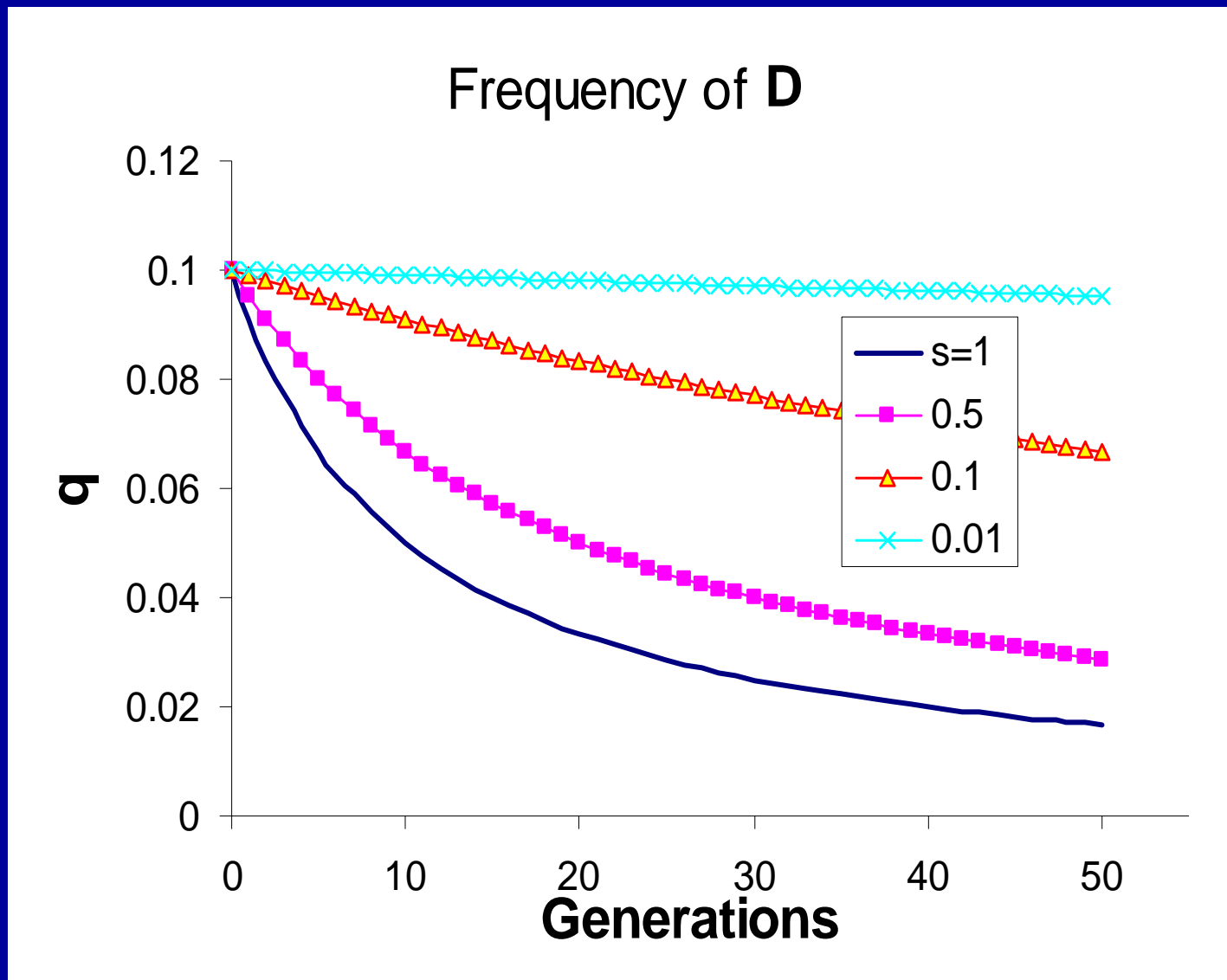
Solution

NN	ND	DD
$(1 - q_0)^2$	$2 q_0 (1 - q_0)$	$(1-s) q_0^2$

- It is easy to show that

$$q_1 = \frac{q_0(1 - q_0) + q_0^2(1 - s)}{(1 - q_0)^2 + 2q_0(1 - q_0) + q_0^2(1 - s)} =$$
$$\frac{q_0(1 - q_0s)}{1 - sq_0^2} = \frac{q_0}{1 + sq_0}$$

Elimination at different s



Mutation and selection

- Two alleles, **D** (deleterious) and **N** (normal)
- Survival in **DD** is decreased by some **s**:
 - only $(1 - s)$ survive and reproduce
- Thus selection eliminates **D** from population

- Opposing force: mutation changes **N** to **D** with probability μ (per gamete generated)
- There must be a mutation / selection balance (equilibrium point)

Fate of a rare recessive mutation

- At the equilibrium point, the frequency of **N** in next generation must be the same as in previous generation

$$p = \frac{(p^2 + pq) \cdot (1 - \mu)}{1 - sq^2}$$

$$p = \frac{p \cdot (1 - \mu)}{1 - sq^2}$$

$$p \cdot (1 - sq^2) = p \cdot (1 - \mu)$$

$$sq^2 = \mu$$

$$q = \sqrt{\frac{\mu}{s}}$$

Problem: cystic fibrosis

- Mutation in CFTR gene
- Recessive monogenic model
- Homozygous carriers ha(ve)(d) $\frac{1}{4}$ chance to die before reproduction age
- Carrier frequency is $\frac{1}{30}$

- Assuming mutation/selection balance model, what is mutation rate, μ ?

Solution

- $P(\mathbf{D})^2 = \mu / \mathbf{s}$
- $\mu = \mathbf{s} P(\mathbf{D})^2$

- $\mathbf{s} = 1/4$
- $P(\mathbf{D}) = 1/(2*30) = 1/60$

- $\mu = 1/4 * 1/3600 = 10^{-5}$
- A bit too high...

Balancing selection

Genotype	AA	Aa	aa
Success	$1 - s_1$	1	$1 - s_2$
Sel. press.	s_1	0	s_2

- Idea: because of heterozygous advantage, both A and a must be present in population

Solution

- Let us denote frequency of "A" as q

- Equation:

$$q' = [q^2 (1 - s_1) + pq] / [1 - s_1 p^2 - s_2 q^2]$$

- At equilibrium point:

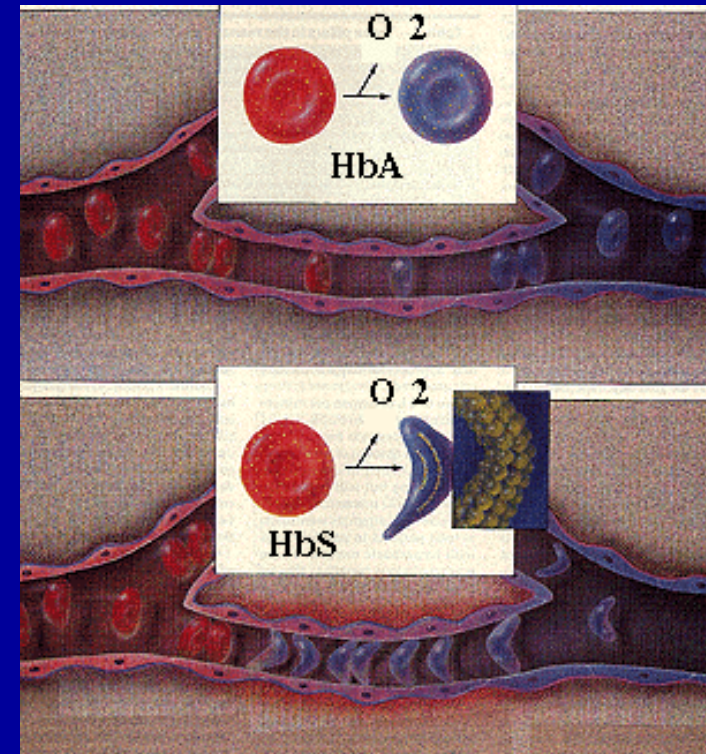
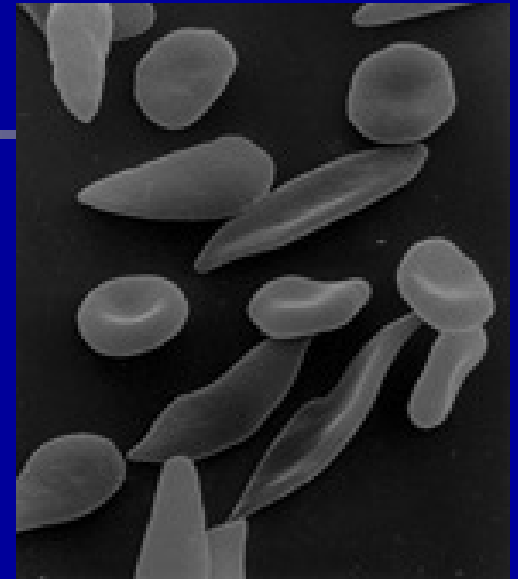
$$q' = q$$

- Solution:

$$q = s_1 / (s_1 + s_2)$$

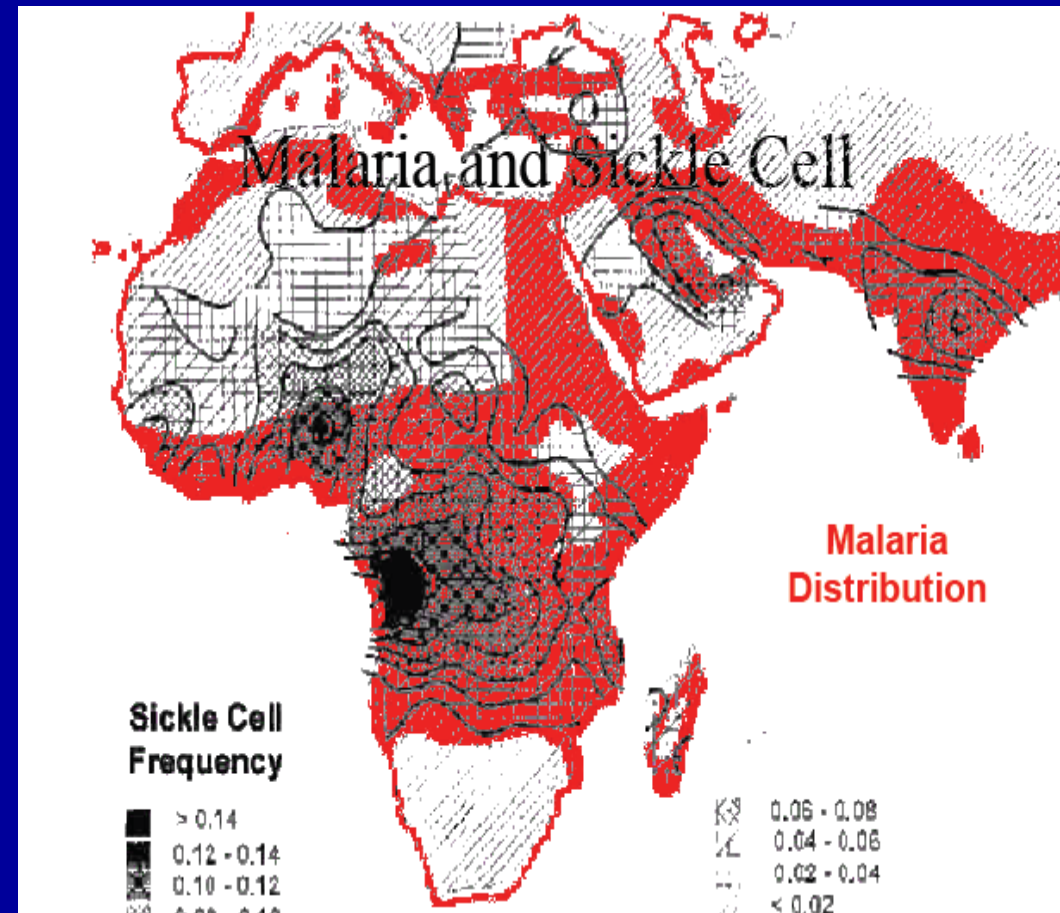
Sickle-cell anemia (SCA)

- HbS allele of human beta-globin
 - Hb/Hb \Rightarrow normal
 - HbS/Hb \Rightarrow normal
 - HbS/HbS \Rightarrow SCA
- SCA
 - sickle cells are "sticky" \Rightarrow vasoocclusion and local hypoxia \Rightarrow vascular damage, organ infarcts, painful crises
 - High risk of death before 3 y.o.
 - Shorter life expectancy in adult



High HbS frequency \leftrightarrow malaria

- Hb/Hb
 - malaria
- HbS/Hb
 - Malaria less frequent, lower parasite level \Rightarrow 30% more survive to adulthood
- HbS/HbS
 - SCA
 - Malaria less frequent, lower parasite level



Application to SCA

- Assume

- Malaria is endemic

Genotype	Hb/Hb	Hb/HbS	HbS/HbS
Survival	0.77	1	0.1
Sel. press.	0.23	0	0.9

- Equilibrium frequency

$$q = 0.23 / (0.23 + 0.9) = 0.20$$

Fitness is context-dependent

A “harmful” allele may

- ... have been favorable or neutral in the past
- ... may be harmful in homo- but favorable in heterozygous form
- ... may have opposite effect at different ages
- ... may have opposite effect on different traits
- ...
- If population is/was small, it also could attain high frequency just by chance (drift)