Genetics of populations

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GE02: day 1 part 3

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Overview

- Subject of populational genetics
- What is population
- Major forces: selection, mutation, drift
- Hardy-Weinberg equilibrium

Subject: Microevolution

Study of genetic changes which happen in populations under influence of evolutionary forces

Given a set of conditions, how frequencies of particular genetic variants will change in time (and space)

What is a “population”?

Two individuals A and B belong to the same genetic population if

- the probability that they would have an offspring in commomn is greater then zero and
- this probability is much higher than the probability of A and B having an offspring in common with some individual C, which is said to be belonging to other population

Island populations

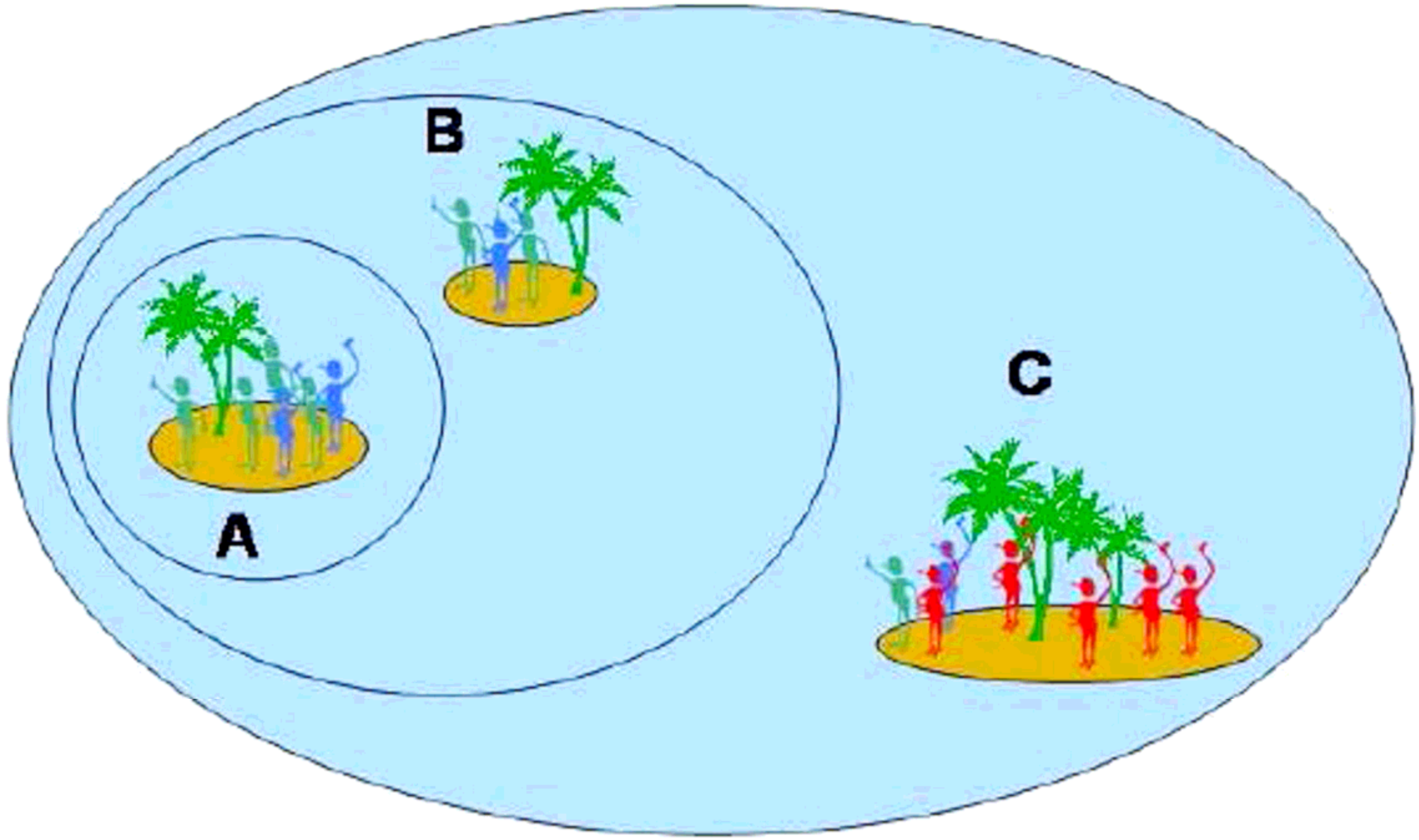

Fig. 1.1. Island populations A, B and C. The isolation is assumed to be proportional to distance, which is relatively small between A and B and large between A, B and C.

Evolutionary forces

Selection is a process of differential reproduction

Mutation is the process in which one allele is changed to other

Random processes, e.g. drift

Genetic processes in large populaions

Assumptions:

- Infinitely large population
- Generation $\Rightarrow$ Gametic pool $\Rightarrow$ Generation
- Random, independent segregation and aggregation of alleles (Mendel's law)

Hardy-Weinberg equilibrium

Consider two alleles, N and D, are segregating in a population. Frequency of D, P(D) = 0.1

If aggregation of alleles is independent and random, what are the expected genotypic proportions?

Solution

Homozygotes

- P(N and N) = P(N) x P(N) = 0.9 x 0.9 = 0.81
- P(D and D) = P(D) x P(D) = 0.1 x 0.1 = 0.01

Heterozygote

- P(N and D) = P(N) x P(D) = 0.9 x 0.1 = 0.09
- P(D and N) = P(D) x P(N) = 0.1 x 0.9 = 0.09

- Total, P(ND or DN) = P(ND) + P(DN) = 0.18

Hardy-Weinberg equilibrium (HWE)

If frequency of allele D is q and the frequency of N is $p = (1 - q)$ then

- $P(DD) = q^2$
- $P(ND) = 2\,p\,q$
- $P(DD) = p^2$

These proportions are known as HWE

P(ND) is termed heterozygosity, a measure of marker informatively

Problem

Consider three alleles, A_1, A_2 and A_3, segregating in a population

- $P(A_1) = 0.1$ and $P(A_2) = 0.2$

Aggregation of alleles is independent and random

What is

- Frequency of A_3?
- How many unordered genotypes can be observed?
- What are equilibrium proportions?

Solution

- Frequency of A_3?

  $P(A_3) = 1 - P(A_1) - P(A_2) = 0.7$

  How many genotypes can be observed?

  - 9 ordered genotypes
  - Six unordered: A_1A_1, A_1A_2, A_1A_3, A_2A_2, A_2A_3, and A_3A_3

  If there are *n* alleles, number of unordered genotypes is $n(n+1)/2$

What are equilibrium proportions?

$$P(A_1A_1) = P(A_1)\,P(A_1) = 0.01$$
$$P(A_1A_2) = 2\,P(A_1)\,P(A_2) = 0.04$$
$$P(A_1A_3) = 2\,P(A_1)\,P(A_3) = 0.14$$
$$P(A_2A_2) = P(A_2)\,P(A_2) = 0.04$$
$$P(A_2A_3) = 2\,P(A_2)\,P(A_3) = 0.28$$
$$P(A_3A_3) = P(A_3)\,P(A_3) = 0.49$$

HWE for multiple alleles

- $P(A_iA_i) = P(A_i)^2$
- $P(A_iA_j) = 2\ P(A_i)\ P(A_j)$

- Heterozygosity is defined as

$$\Sigma_{i > j}\ 2\ P(A_i)\ P(A_j)$$

When HWE is reached?

If the frequency of genotypes are

- P(DD) = 0.1, P(ND) = 0.2 and P(NN) = 0.7

Questions

- What is the frequency of D, P(D)?
- What will be genotypic frequencies after a generation of random mating?

Follow the model…

- What is the frequency of D, P(D)?
  - P(D) = q = P(DD) + P(ND)/2 = 0.1 + 0.2/2 = 0.2
- Now the gametes start randomly aggregate:

	Allele	N	D
Allele	Freq	0.8	0.2
N	0.8	0.64	0.16
D	0.2	0.16	0.04

  - P(DD) = 0.04, P(ND) = 0.32, P(NN) = 0.64
  - This follows HWE with q=0.2

More general

With any initial conditions, denote $q = P(DD) + P(ND)/2$

After a round of random mating:

	Allele	N	D
Allele	Freq	$(1-q)$	q
N	$(1-q)$	$(1-q)^2$	$(1-q)q$
D	q	$(1-q)q$	q^2

In next generation, P(D) is $q' = q^2 + 2q(1-q)/2 = q$

- Under HWE, allelic frequencies stay stable over time
- HWE is reached after one generation of random mating

Frequency of carriers

- Bi-allelic system: N and D, $P(D) = q$
- Carriers of D: ND or DD
- $P(ND \text{ or } DD) = 2pq + q^2$
- When $q \rightarrow 0$ [and $p = (1-q) \rightarrow 1$]

$$P(ND + DD) \sim 2q$$

Goodness of approximation