

Selection

Mutation & selection balance

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GE02: day 2 part 1

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Selection

- Acts through differential reproduction of individuals having different genotypes
- It is assumed that some genotypes are less “fit” than others
- Fitness of some genotype, g , is expressed as probability of reproductive success of an individual having this genotype

$$r(g) = 1 - s(g)$$

where $s(g)$ is probability that a person with genotype g will fail to reproduce

$r(g)$ is some times termed “**R**eproductive success” or “survival”

$s(g)$ is some times termed “**S**election pressure” or “mortality”

Differential reproduction

- Can occur because of different mechanisms
 - Survival, e.g. a mutation may lead to death before reproductive age
 - Damaged reproductive system, e.g. complete sterility or lowered fertility
 - Distorted sexual behavior
 - ...

Problem: dominant lethal allele

- Two alleles are present in population, **A** and **a**
- Initial frequency of **a** is p_0
- **AA** have normal fitness while carriers of **a** have high mortality (are dead before birth):
$$s(\mathbf{AA}) = 1, s(\mathbf{Aa}) = s(\mathbf{aa}) = 0$$
- What will happen to the **a** allele after some time?

Solution

- It will be immediately eliminated!

Problem: recessive model

- Two alleles are present in population, **A** and **a**
- Initial frequency of **a** is p_0
- Population is large and mating is random
- Individuals of genotype **aa** are sterile (do not mate):

$$s(\mathbf{AA}) = s(\mathbf{Aa}) = 0; s(\mathbf{aa})=1$$

- What will happen to the **a** allele after some time?

Solution

- Genotypic frequencies in 1st generation:

$$(1 - p_0)^2 : 2 p_0 (1 - p_0) : p_0^2$$

- Only **AA** and **Aa** contribute to the next generation
- New frequency will be

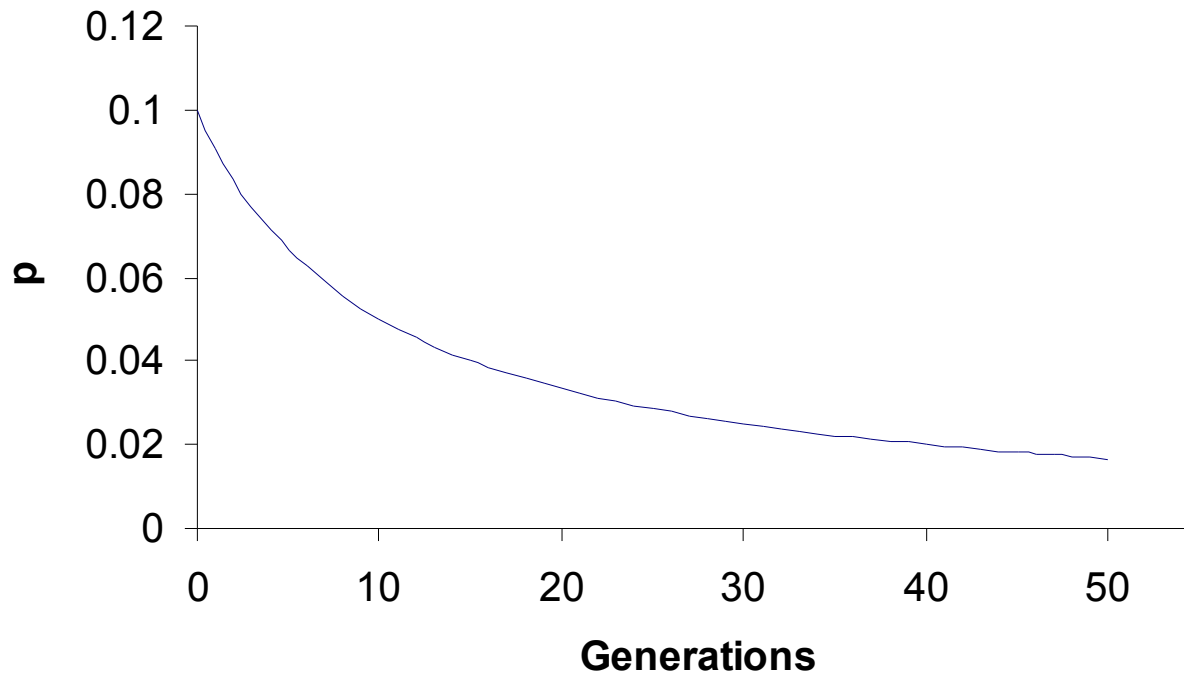
$$p_1 = p_0 (1 - p_0) / (1 - p_0^2)$$

- Next generation

$$p_2 = p_1 (1 - p_1) / (1 - p_1^2)$$

Elimination of mutant

Frequency of "a"



Fate of a rare recessive mutation

- Two alleles, M (deleterious) and N (normal)
- Mutation changes N to M with probability μ
- Survival in MM is affected, say only $(1 - s)$ survive and reproduce (for previous example, s was 1)
- There must be mutation / selection balance

Fate of a rare recessive mutation

- At equilibrium point, the frequency of N in next generation must be the same as in previous generation

$$P(N) = \frac{(P(N) \cdot P(M) + P(N)^2) \cdot (1 - \mu)}{1 - s P(M)^2} = \frac{P(N) \cdot (1 - \mu)}{1 - s P(M)^2}$$

- This gives the equilibrium solution

$$P(M)^2 = \mu/s$$

Example: cystic fibrosis

- Mutation in CFTR gene
- Recessive monogenic mode
- Homozygous carriers has $\frac{1}{4}$ chance to die before reproduction age
- Carrier frequency is $\frac{1}{30}$
- Assuming mutation/selection balance model, what is mutation rate, μ ?

Solution

- $P(M)^2 = \mu/s$
- $\mu = s P(M)^2$
- $s = 1/4$
- $P(M) = 1/(2*30) = 1/60$
- $\mu = 1/4 * 1/3600 = 10^{-5}$ (a bit too high...)

Balancing selection

Genotype	AA	Aa	aa
Survival	$1 - s_1$	1	$1 - s_2$
Sel. press.	s_1	1	s_2

- Idea: because of heterozygous advantage, both A and a must be present in population

Solution

- Equation:

$$q' = [q^2 (1 - s_1) + pq] / [1 - s_1 p^2 - s_2 q^2]$$

- At equilibrium point:

$$q' = q$$

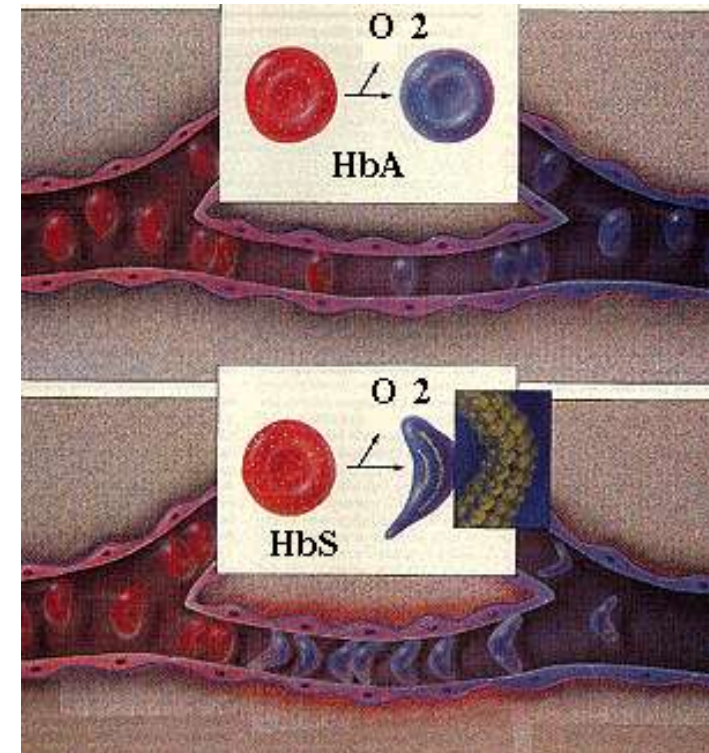
- Solution:

$$q = s_1 / (s_1 + s_2)$$

Sickle-cell anemia (SCA)

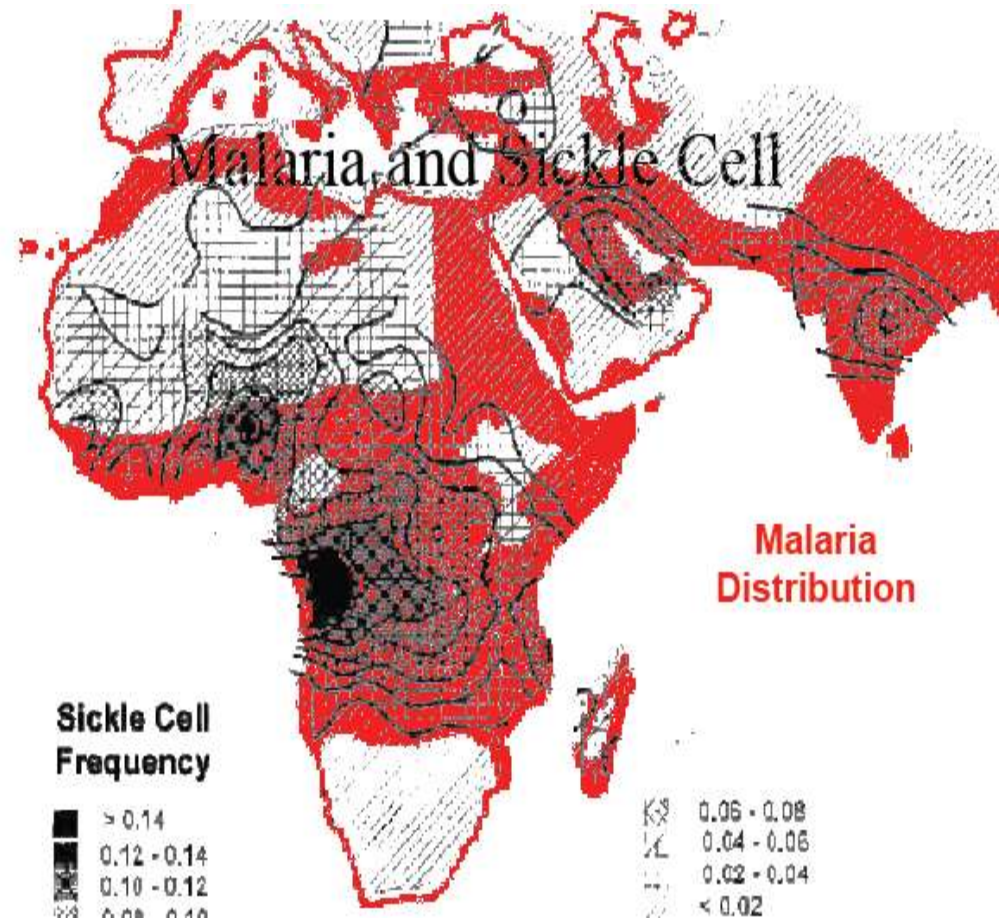


- HbS allele of human beta-globin
 - Hb/Hb => normal
 - HbS/Hb => normal
 - HbS/HbS => SCA
- SCA
 - sickle cells are “sticky” => vaso-occlusion and local hypoxia => vascular damage, organ infarcts, painful crises
 - High risk of death before 3 y.o.
 - Shorter life expectancy in adult



High HbS frequency \leftrightarrow malaria

- Hb/Hb
 - malaria
- HbS/Hb
 - Malaria less frequent, lower parasite level \Rightarrow 29% more survive to adulthood
- HbS/HbS
 - SCA
 - Malaria less frequent, lower parasite level



Application to SCA

- Assume

- Malaria is endemic

Genotype	AA	Aa	aa
Survival	0.78	1	0.2 (?)
Sel. press.	0.22	1	0.8

- Equilibrium frequency

$$q = 0.22 / (0.22 + 0.8) = 0.216$$

The lesson to learn

- Fitness of a genotype is context-dependent, a “harmful” allele may
 - ... have been favorable or neutral in the past
 - ... may be harmful in homo- but favorable in heterozygous form
 - ... because of stochastic processes (drift, to be considered later)