## Q1 Color-blindness (Feller, Ch.5, Ex. 7)

Let us assume that the frequencies of color-blindness are $5 \%$ and $0.25 \%$ of in men and women, respectively.
Question: What is the chance that a given color-blind person is a man?
Answer
$0.05 /(0.05+0.0025)=95 \%$

## Q2 Testing HWE

Consider a SNP with variants, A and T. In an association analysis experiment, the following data were obtained: 9 subjects had genotype AA, 136 subjects had genotype AT and 392 subjects had genotype TT.
Question: what is the frequency of A allele? Do the genotypic frequencies follow Hardy-Weinberg distribution?

Answer
The frequency of A allele may be estimated using exoression

$$
\mathrm{P}(\mathrm{~A})=[2 \mathrm{~N}(\mathrm{AA})+\mathrm{N}(\mathrm{AT})] /[2 \mathrm{~N}]=[29+136] /[2537]=0.14338919925512
$$

Then, if Hardy-Weinberg equilibrium is true, expected frequencies
of genotypes are

$$
\begin{aligned}
& \mathrm{P}(\mathrm{AA})=\mathrm{P}(\mathrm{~A}) \mathrm{P}(\mathrm{~A})=0.020560462463025 \\
& \mathrm{P}(\mathrm{AT})=2 \mathrm{P}(\mathrm{~A}) \mathrm{P}(\mathrm{~T})=2 \mathrm{P}(\mathrm{~A})(1-\mathrm{P}(\mathrm{~A}))=0.24565747358419 \\
& \mathrm{P}(\mathrm{TT})=\mathrm{P}(\mathrm{~T}) \mathrm{P}(\mathrm{~T})=(1-\mathrm{P}(\mathrm{~A}))(1-\mathrm{P}(\mathrm{~A}))=0.73378206395278
\end{aligned}
$$

and the expected number of people with these genotypes are $11.040968342644,131.91806331471,394.04096834264$
in AA, AT and TT groups, respectively
The chi-square statistics is calculated to test is actual data fit
HWE expectation:

$$
\begin{aligned}
& \text { CHI } 2=\text { SUM }(\mathrm{OBS}-\mathrm{EXP})^{\wedge} 2 / \mathrm{EXP}= \\
& \begin{array}{l}
(9-11.040968342644)^{\wedge} 2 / 11.040968342644+ \\
(136-131.91806331471)^{\wedge} 2 / 131.91806331471+ \\
\\
0.51415999216947 \quad(392-394.04096834264)^{\wedge} 2 / 394.04096834264=
\end{array}
\end{aligned}
$$

This CHI2 has one degree of freedom, consequently, if CHI $2>3.84$, the null hypothesis of HWE is to be rejected.
In this case $0.51415999216947<3.84$ and, consequently, the null hypothesis holds.

## Q3 HWE with multiple alleles

Consider a locus with 3 alleles (named from A_1 to A_3). The frequency of allele A_1 ... A_2 are $0.6713,0.1861$, respectively.
Question: What are the frequencies of $A_{-} 1 / A_{-} 1, A_{-} 1 / A_{-} 2, \ldots$ A_3/A_3 genotypes under Hardy-Weinberg equilibrium?
Answer
The frequencies of alleles are
$\operatorname{Pr}\left(A \_1\right)=0.6713$
$\operatorname{Pr}(\mathrm{A}$ 2) $)=0.1861$
$\operatorname{Pr}(\mathrm{A}$ _3) $=0.1426$
According to Hardy-Weinberg principle,
$\operatorname{Pr}\left(A \_1 / A \_1\right)=\operatorname{Pr}\left(A \_1\right) * \operatorname{Pr}\left(A \_1\right)=0.6713 * 0.6713=0.45064369$
$\operatorname{Pr}\left(A \_1 / A \_2\right)=2 * \overline{\operatorname{Pr}}(\mathrm{~A} 1) * \operatorname{Pr}\left(\mathrm{~A} \_2\right)=2 * 0.6713 * 0.1861=0.24985786$
$\operatorname{Pr}\left(\mathrm{A} \_1 / \mathrm{A} \_3\right)=2 * \operatorname{Pr}\left(\mathrm{~A} \_1\right) * \operatorname{Pr}\left(\mathrm{~A} \_3\right)=2 * 0.6713 * 0.1426=0.19145476$
$\operatorname{Pr}\left(\mathrm{A} \_2 / \mathrm{A} \_2\right)=\operatorname{Pr}\left(\mathrm{A} \_2\right) * \operatorname{Pr}\left(\mathrm{~A} \_2\right)=0.1861 * 0.1861=0.03463321$
$\operatorname{Pr}\left(\mathrm{A} \_2 / \mathrm{A} \_3\right)=2 * \operatorname{Pr}\left(\mathrm{~A} \_2\right) * \operatorname{Pr}\left(\mathrm{~A} \_3\right)=2 * 0.1861 * 0.1426=0.05307572$
$\operatorname{Pr}\left(\mathrm{A} \_3 / \mathrm{A} \_3\right)=\operatorname{Pr}(\mathrm{A} 3)^{*} \operatorname{Pr}\left(\mathrm{~A} \_3\right)=0.1426 * 0.1426=0.02033476$

## Q4 X-linked recessive disease

For pedigree $\mathbf{A}$ : what is the probability that " i " is a carrier? What is the risk for her next boy-child?
For pedigree $\mathbf{B}$ : what is the probability that " $h$ " is a carrier? What is the risk for her next boy-child?


Answer:
A: the chance $g$ is carrier is $1 / 2$. The chance $i$ is carrier is $1 / 4$.
$B$ : the a priori chance for $h$ to be carrier is $1 / 2$. To incorporate information on $j$ and $k$, use Bayes. The posterior probability is $\mathbf{1 / 5}$. The risk for next boy-child is $1 / 10$.

The same question for the following pedigree, person A


Answer Q4 v. 1
Q4.1.1 You have to take step-wise approach to this problem. First, the posterior probability that grandmother is carrier is to be calculated using information that (1) the prior probability for her to be a carrier is $1 / 2$ and (2) information that she has 3 healthy boy-children. Next, half of the posterior makes prior for MOTHER of A to be carrier. Based on this information + information that A has 4 healthy brothers, posterior probability that mother of A is to be calculated. Half of this is used as prior for A to be carrier. After incorporation of the information that A already has one healthy boy-child, the answer is ready.

For grandmother of A:

| Hypothesis | Carrier | Non- <br> carrier |  |
| :--- | :--- | :--- | :--- |
| Prior | $1 / 2$ | $1 / 2$ |  |
| Conditional | $1 / 2^{3}$ | 1 |  |
| Joint | $1 / 2^{4}$ | $1 / 2$ | $9 / 1$ |
| Posterior | $1 / 9$ | $8 / 9$ |  |


| For mother of A |  |  |  |
| :--- | :--- | :--- | :--- |
| Hypothesis | Carrier | Non- <br> carrier |  |
| Prior | $1 / 18$ | $17 / 18$ |  |
| Conditional | $1 / 2^{4}$ | 1 |  |
| Joint | $1 / 288$ | $17 / 18$ | $273 / 28$ <br> 8 |
| Posterior | $1 / 273$ | $272 / 27$ |  |
|  |  | 3 |  |

For A

| Hypothesis | Carrier | Non-carrier |  |
| :--- | :--- | :--- | :--- |
| Prior | $1 / 546$ | $545 / 546$ |  |
| Conditional | $1 / 2$ | 1 |  |
| Joint | $1 / 1092$ | $545 / 546$ | $1091 / 1092$ |
| Posterior | $1 / 1091$ | $1090 / 1091$ |  |

## Thus the probability that $\mathbf{A}$ is carrier is $\mathbf{1} / \mathbf{1 0 9 1}=\mathbf{0 . 0 0 0 9 2}$

## Q5 Dominant disease

Given a dominant disease with allele frequency of 0.03 , what is the risk for next child?


Answer:
Probability given parent's phenotype, $\mathrm{P}(\mathrm{MN} \mid$ affected $)$ :

|  | 0,0300 | DN | DD |  |
| :--- | :--- | :--- | :--- | :--- |
| prior | 0,0582 | 0,0009 |  |  |
| cond | 1,0000 | 1,0000 Total |  |  |
| joint | 0,0582 | 0,0009 | 0,0591 |  |
| post |  | 0,9848 | 0,0152 |  |

Probability given 3 affected offspring, $\mathrm{P}(\mathrm{MN} \mid 3 \times \mathrm{D}$, also parent $=\mathrm{D})$

|  | DN | DD |  |
| :--- | ---: | :--- | :--- |
| prior | 0,9848 | 0,0152 |  |
| cond | 0,1250 | 1,0000 Total |  |
| joint | 0,1231 | 0,0152 | 0,1383 |
| post | 0,8899 | 0,1101 |  |

Risk for a next child $=1 / 2 P(M N \mid 3 \times D$, also parent $=D)+P(M M \mid 3 \times D$, also parent $=D)=0.555$

## Q5 Receswsive disease, more information

Considered pedigree with a recessive condition. Assume that the frequency of mutation in the population is very low, $0.2 \%$. What is the risk for the "?" child? How it compares with populational risk of the disease (compute RR)? If both parents test negatively at the test with $70 \%$ sensivity, how this modifies the risk prediction and RR?


Answer:
Based on phenotype only,
for father $\mathrm{P}(\mathrm{MN} \mid$ data $)=2 / 3$
for mother $\mathrm{P}(\mathrm{MN} \mid$ data $)=1 / 4$
Use these posterior as prior utilizing the test information and consider hypotheses "both are MN" and "NOT"

|  | both MN | NO |  |
| :--- | ---: | :--- | :--- |
| prior | 0,1667 | 0,8333 |  |
| cond | 0,1500 | 1,0000 Total |  |
| joint | 0,0250 | 0,8333 | 0,8583 |
| post | 0,0291 | 0,9709 |  |

Risk for the child is $0.0291 \times 1 / 4=0.0073$

